Logarithms Example

Use logarithm properties, respect the domain, and verify the valid solution.

Solve: log base 2 of (x - 1) + log base 2 of (x - 3) = 3

log_b(M) + log_b(N) = log_b(MN); log_b(A) = c means A = b^c

Step-by-step solution

1. Set the domain

x - 1 > 0 and x - 3 > 0, so x > 3.

2. Combine the logarithms

log base 2 of ((x - 1)(x - 3)) = 3.

3. Convert to exponential form

(x - 1)(x - 3) = 2^3 = 8.

4. Solve the quadratic

x^2 - 4x + 3 = 8; x^2 - 4x - 5 = 0; (x - 5)(x + 1) = 0.

5. Apply the domain restriction

Candidates are x = 5 and x = -1. Since x > 3, only x = 5 is valid.

Answer

x = 5

Check

log base 2 of 4 + log base 2 of 2 = 2 + 1 = 3. Both arguments are positive, so x = 5 checks.