Modular Arithmetic Example

Add 38 and 27 modulo 7

WORKED EXAMPLE

Modular arithmetic compares remainders after division by a fixed positive integer, called the modulus. Congruent numbers have the same remainder modulo that integer.

Problem

Find (38 + 27) mod 7.

KEY FACT	VALUE
Modulus	7
38 mod 7	3
27 mod 7	6
Least nonnegative residue	2

Step-by-step solution

1. Reduce the first addend

Since 7 x 5 = 35, dividing 38 by 7 leaves remainder 3.

38 = 7 x 5 + 3; 38 is congruent to 3 mod 7

2. Reduce the second addend

Since 7 x 3 = 21, dividing 27 by 7 leaves remainder 6.

27 = 7 x 3 + 6; 27 is congruent to 6 mod 7

3. Add the remainders

Addition preserves congruence. The remainder sum 9 is congruent to 2 modulo 7.

38 + 27 is congruent to 3 + 6 = 9 is congruent to 2 mod 7

4. State the standard residue

The least nonnegative residue is the value from 0 through 6 that is congruent to the original sum.

(38 + 27) mod 7 = 2

ANSWER

(38 + 27) mod 7 = 2.

Check

The original sum is 65. Dividing by 7 gives 65 = 7 x 9 + 2, so the remainder is 2.