Patterns Example

Find a term and partial sum in an arithmetic sequence with a constant difference.

A number pattern is an ordered list generated by a rule. In an arithmetic sequence, consecutive terms differ by the same amount, called the common difference. The nth-term formula finds a distant term without listing every term; the arithmetic-series formula finds the sum of the first n terms. These are standard results from the study of progressions, not discoveries attributed to one inventor.

FORMULA

$$\text{Arithmetic term: } a_n = a_1 + (n-1)d; \text{ sum: } S_n = n(a_1+a_n)/2.$$

WORKED EXAMPLE

For 4, 7, 10, 13, ... find the 12th term and the sum of the first 12 terms.

1. Find the common difference

Subtract consecutive terms. Each step adds 3, so $a_1 = 4$ and $d = 3$.

$$7-4 = 3;\ 10-7 = 3;\ 13-10 = 3$$

2. Use the nth-term formula

Substitute $n = 12$, $a_1 = 4$, and $d = 3$. There are 11 jumps to the 12th term.

$$a_{12} = 4 + (12-1) \times 3 = 37$$

3. Use the arithmetic-series formula

Multiply the number of terms by the average of the first and last terms.

$$S_{12} = 12(4+37)/2 = 246$$

$$a_{12} = 37;\ S_{12} = 246$$

The 12th term is 37, and the sum of the first 12 terms is 246.

CHECK

Pair first with last, then move inward. There are six pairs, each summing to 41.

$$(4+37)+(7+34)+\ldots+(19+22) = 6 \times 41 = 246$$

Remember: Use $a_n = a_1 + (n-1)d$ for a term. Use $S_n = n(a_1+a_n)/2$ for the sum of the first n terms.