Factor a Quadratic and Find Its Roots

A quadratic equation has degree 2 and standard form $ax^2 + bx + c = 0$. Factoring rewrites the trinomial as a product. For a monic quadratic, find two numbers whose sum is b and product is c; the zero-product property then gives the roots. This example verifies both roots by substitution.

FORMULA

For $x^2 + bx + c$, find r and s with $r + s = b$ and $rs = c$. If $uv = 0$, then $u = 0$ or $v = 0$.

WORKED EXAMPLE

Solve $x^2 - 5x + 6 = 0$ by factoring.

1. Find a factor pair for 6

The constants must multiply to 6. The pair −2 and −3 also adds to −5, the coefficient of x.

$$(-2)(-3) = 6;\ (-2) + (-3) = -5$$

2. Factor the trinomial

Use the pair as the constants in the binomial factors.

$$x^2 - 5x + 6 = (x - 2)(x - 3)$$

3. Apply the zero-product property

A product is zero only if at least one factor is zero. Solve each factor equation.

$$x - 2 = 0 \text{ or } x - 3 = 0;\ \text{therefore } x = 2 \text{ or } x = 3$$

ANSWER

Roots: x = 2 and x = 3

The factored equation is $(x - 2)(x - 3) = 0$. Each factor gives one root.

CHECK

At x = 2: $4 - 10 + 6 = 0$. At x = 3: $9 - 15 + 6 = 0$.