Rational Expressions Example

Simplify a quotient of polynomials by factoring first, canceling common factors, and preserving values excluded by the original denominator.

WORKED EXAMPLE

Simplify (x^2 - 9)/(x^2 + 5x + 6).

Difference of squares: a^2 - b^2 = (a - b)(a + b)

Step-by-step solution

1. Factor numerator and denominator. The numerator is a difference of squares. For the denominator, 2 and 3 multiply to 6 and add to 5.

x^2 - 9 = (x - 3)(x + 3); x^2 + 5x + 6 = (x + 2)(x + 3)

2. Record excluded values before canceling. The original denominator is zero when x = -2 or x = -3.

x^2 + 5x + 6 = 0; exclude x = -2 and x = -3

3. Cancel the common factor. Cancel x + 3, but keep x = -3 excluded because it made the original denominator zero.

[(x - 3)(x + 3)]/[(x + 2)(x + 3)] = (x - 3)/(x + 2)

ANSWER

(x - 3)/(x + 2), where x != -2 and x != -3

Check at x = 1. Both forms equal -2/3. The original restrictions still apply.

(1^2 - 9)/(1^2 + 5(1) + 6) = -8/12 = -2/3; (1 - 3)/(1 + 2) = -2/3

Remember: cancel common factors, never individual terms across addition or subtraction.