Roots Example

Simplify a square root by separating out the largest perfect-square factor.

A square root reverses squaring: sqrt(n) is the nonnegative number whose square is n. To simplify a radical, factor its radicand into a perfect square and a remaining factor. The product rule applies to nonnegative factors. This example keeps the exact radical separate from its decimal approximation.

FORMULA

sqrt(a x b) = sqrt(a) x sqrt(b), for a,b >= 0; sqrt(k^2 x m) = k x sqrt(m), for k >= 0.

WORKED EXAMPLE

Simplify sqrt(72).

1. Find a perfect-square factor

The largest perfect-square factor of 72 is 36, leaving 2.

72 = 36 x 2

2. Separate the product under the root

Since sqrt(36) = 6, move 6 outside the radical.

sqrt(72) = sqrt(36 x 2) = sqrt(36) x sqrt(2) = 6sqrt(2)

3. Give an optional decimal estimate

The radical form is exact. Use sqrt(2) ~= 1.4142 for an estimate.

6sqrt(2) ~= 6 x 1.4142 ~= 8.4852

sqrt(72) = 6sqrt(2) ~= 8.4852

The exact simplified radical is 6sqrt(2); 8.4852 is an approximation.

CHECK

Squaring the exact result recovers the original radicand. The principal square root is nonnegative.

(6sqrt(2))^2 = 36 x 2 = 72

Remember: A radical is simplified when no perfect-square factor greater than 1 remains inside the square root.